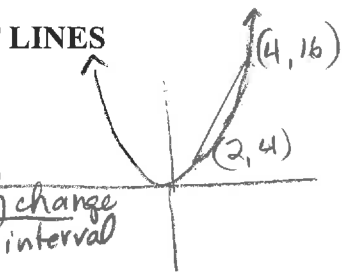
AP Calculus 42S **2.4 RATES OF CHANGE AND TANGENT LINES**



Average Rates of Change (Review)

Sketch $f(x)=x^2$. Draw a secant line between 2 points.

Slope of secant = average rate of change. i.e. $\frac{\Delta y}{\Delta x} = \frac{\text{amount of change}}{\text{length of interval}}$

Ex. (x, x^2)

from $(2,4)$ to $(4, 4^2)$ $\quad m = \frac{16-4}{4-2} = 6$

$(2,4)$ to $(3, 3^2)$ $\quad m = \frac{9-4}{3-2} = 5$

$(2,4)$ to $(2.5, 2.5^2)$ $\quad m = \frac{6.25-4}{2.5-2} = 4.5$

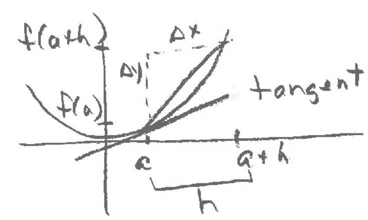
i.e. $(2,4)$ to $(2+h, (2+h)^2)$ $\quad m = \frac{(2+h)^2-4}{2+h-2} = \frac{4+4h+h^2-4}{h}$

$= \frac{h^2+4h}{h} = h+4$

$\lim_{h \to 0}(h+4) = \boxed{4}$

Average rate of change = slope of secant at "a" = $\frac{f(a+h)-f(a)}{h} = \frac{\Delta y}{\Delta x}$

Instantaneous Rate of Change (touched on this in 2.1)



Slope of tangent = instantaneous rate of change

Slope of tangent at "a" = $\lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$

See p.85 Fig. 2.30

Tangent to a curve is found by limiting the value of the secant slope as interval h gets smaller and smaller.

The tangent line to the curve at P is the line through P with slope = $\lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$

Ex. $y = \frac{1}{x}$ at $\left(a, \frac{1}{a}\right)$ (x, y) $\quad m_{tan} = \lim_{h \to 0} \frac{\frac{1}{a+h} - \frac{1}{a}}{h} = \lim_{h \to 0} \frac{\frac{a-(a+h)}{a(a+h)}}{\frac{h}{1}}$

$= \lim_{h \to 0} \frac{-h}{a(a+h)} \cdot \frac{1}{h} = \lim_{h \to 0} \frac{-1}{a(a+h)} = \boxed{-\frac{1}{a^2}}$

Suppose $a=2$ i.e point $(2, \frac{1}{2})$ $\quad m_{tan} = -\frac{1}{a^2} = -\frac{1}{2^2} = -\frac{1}{4}$

Eqn: $\boxed{y - \frac{1}{2} = -\frac{1}{4}(x-2)}$ or $y = -\frac{1}{4}x + 1$

The normal line to a curve at a point is the line perpendicular to the tangent at that point.

Ex. $y = \frac{1}{x}$ at $\left(2, \frac{1}{2}\right)$ $\quad m_{\perp} = \frac{-1}{m_{tan}}$ $\quad m_{\perp} = 4$

Eqn: $\boxed{y - \frac{1}{2} = 4(x-2)}$ or $y = 4x - \frac{15}{2}$

Slope of secant (at $x = a$) (Review!)

$$\frac{f(a+h) - f(a)}{h} \quad \text{or} \quad \frac{f(b) - f(a)}{b-a}$$

Slope of tangent (at $x = a$)

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Examples: (Continue in your notebook.) or at the board. p. 87 #2, p. 88 #10, 12, 14, 16, 22

Start Assignment: 2.4 p.87 #1-9 odds

Speed

Recall:

Slope of d-t graph is velocity (or speed).

(Speed is always positive, but velocity is positive or negative because it includes direction.)

Slope of v-t graph is acceleration.

Speed = instantaneous rate of change of position with respect to time

$$= \lim_{h \to 0} \frac{f(t+h) - f(t)}{h}$$

Formulas for area and volume: See Appendix A1 p. 577

Examples: (Continue in your notebook.) p. 88 # 24, 26
p. 89 # 30, 32

Assignment: 2.4 p.87 #1-33 odds (leave out trig and log questions)